The National Strategies
Early Years

Children thinking mathematically: PSRN essential knowledge for Early Years practitioners

department for
children, schools and families

Children thinking mathematically: PSRN essential knowledge for Early Years practitioners

First published in 2009

Ref: 00861-2009BKT-EN

Disclaimer

The Department for Children, Schools and Families wishes to make it clear that the Department and its agents accept no responsibility for the actual content of any materials suggested as information sources in this publication, whether these are in the form of printed publications or on a website.

In these materials, icons, logos, software products and websites are used for contextual and practical reasons. Their use should not be interpreted as an endorsement of particular companies or their products.

The websites referred to in these materials existed at the time of going to print.

Please check all website references carefully to see if they have changed and substitute other references where appropriate.

DSI CMM 12-2009

Contents

00861-2009BKT-EN

1. Introduction

Problem Solving, Reasoning and Numeracy in the Early Years Foundation Stage

This booklet focuses on children's mathematical development, which is explored in the area of learning and development entitled Problem Solving, Reasoning and Numeracy (PSRN) in the Early Years Foundation Stage (EYFS) (DCSF, 2008).

The EYFS recognises that *creativity and critical thinking* are important in all areas of learning and are as integral to mathematics as they are to painting or dance. Reflecting on the narratives and examples of children's mathematics throughout this booklet, it is clear that creativity plays a significant role in mathematical thinking and understanding.

The four themes of the EYFS, A Unique Child, Positive Relationships, Enabling Environments and Learning and Development, underpin everything that practitioners do with children from birth to five. They also underpin every chapter in this booklet.

The *Practice Guidance for the Early Years Foundation Stage* (page 63) and the PSRN card remind practitioners that A Unique Child will 'seek patterns, make connections and recognise relationships through finding out about and working with numbers and counting, with sorting and matching and with shape, space and measures' and that 'children use their knowledge and skill in these areas to solve problems, generate new questions and make connections across other areas of Learning and Development.'

The EYFS guidance then goes on to set out some of the important considerations for children's learning in PSRN relating to Positive Relationships, Enabling Environments and Learning and Development. Reference is made to these throughout this booklet.

What mathematics do you think children are exploring here? How could you enhance this?

Why do we need more guidance on PSRN?

The independent review of mathematics teaching in Early Years settings and primary schools, known as the Williams review (DCSF, 2008a), recommended that the DCSF should commission a set of materials on mathematical mark making and children's mathematical development to be used to support Early Years practitioners' professional development.

The review emphasised that although 'the EYFS provides guidance on developing mathematical understanding through imaginative play…opportunities in this area seem to be missed.'

This finding is confirmed by the Early Years Foundation Stage Profile (EYFSP) results, which demonstrate that relatively few children (36 per cent) attain point 8: *uses developing mathematical ideas and methods to solve practical problems,* in any of the three mathematical assessment scales.

The EYFSP scores also reveal that boys do less well than girls especially in the area of calculation: 'In practical activities and discussion, begins to use the vocabulary involved in adding and subtraction' (EYFSP). It is essential therefore that practical opportunities are available on a regular basis within small group activities and throughout the play environment, and that these activities are built on (see example below).

Developing calculation

String beans and containers were set on the carpet for 'talk time' and Brandon (4 years 3 months) decided to shell beans and put them in a container which he had chosen. He self-initiated his own counting. Quietly talking to himself he divided the beans into each section of the container, '11 more now mmm…how many more? 5 now' as he counted the empty spaces. Brandon was using counting to calculate how many more he needed. Brandon's teacher regularly puts out quantities of objects to talk

about and involves the group in talking about increasingly larger quantities. Mathematical vocabulary is deliberately used but in meaningful contexts to give the children tools to articulate their knowledge of mathematics.

Building on *Mark Making Matters*

Mark Making Matters (DCSF, 2008b) was the first part of the set of materials recommended by Williams. It aimed to raise awareness of the importance of young children's mark making as a tool for communication and thinking across the six areas of learning and development, while strengthening the quality of provision for mark making in Communication, Language and Literacy and in PSRN.

A local education authority advisor, involved in raising the profile of calculation through supporting reception class teachers in their understanding of children thinking mathematically through their own graphics, stated, 'Maths is much higher profile now. Teachers and practitioners are talking about calculating significantly more; boys are representing mathematics and are extending themselves much more. This has had an impact on the way in which teachers and practitioners work: their expectations of the children are much higher.'

The Williams review (DCSF, 2008a) also stressed that effective Early Years pedagogy should value and support children's own *mathematical graphics*. Children's own *mathematical graphics* help them understand the written language of mathematics and how it can be used.

What's in this booklet?

This booklet takes *Mark Making Matters* further and extends and develops the concepts explored there with particular reference to the three strands of PSRN:

- Numbers as labels and for counting
- Calculating
- Shape space and measures.

There are chapters on each of these, as well as the opening chapter on enabling environments for PSRN. Chapter 3: *Children's mathematical graphics* lays the foundation for the remainder of the booklet, which concludes with consideration of transition between EYFS and Year 1.

High-quality experiences of mathematics are the entitlement of every child

The examples in this booklet include children from a range of backgrounds and communities. They have different learning abilities (some with identified special needs and some who are gifted) and include children learning English as an additional language. The examples show how all young children have an amazing ability to understand and will often surpass practitioners' expectations to reach their potential – provided they can explore mathematics through play and in personally meaningful ways.

Practitioners need to provide opportunities for all children, ensuring that those less likely to achieve well are fully engaged to support them in developing positive dispositions to learning and realising better outcomes.

This booklet aims to help practitioners 'see' the mathematics in children's play. It is underpinned by the rich legacy of research into young children's mathematical development, play and learning, and illuminates the sorts of context and adult 'scaffolding' that make these experiences so successful.

00861-2009BKT-EN

2. Enabling environments

'Children's play reflects their wide-ranging and varied interests and preoccupations. **In their play children learn at their highest level**'

DfES: 2007b

The EYFS requires that:

Children must be supported in developing their understanding of Problem Solving, Reasoning and Numeracy in a broad range of contexts in which they can explore, enjoy, learn, practise and talk about their developing understanding. They must be provided with opportunities to practise these skills and to gain confidence and competence in their use.

The environment (including the indoor, outdoor and emotional environments) that practitioners provide plays a significant role in supporting young children's mathematical learning. The EYFS makes it clear that young children learn best through play and that for their learning to be effective, they need sensitive and informed support from adults.

All children can be successful with mathematics, provided that they have opportunities to explore mathematical ideas in ways that make personal sense to them and opportunities to develop mathematical concepts and understanding. Children need to know that practitioners are interested in their thinking, respect their ideas, are sensitive to their feelings and value their contributions.

From birth, babies are keen explorers of the world and need a close bond with their mother, father and/or other key adults to give them the confidence to continue as confident learners. This 'secure attachment' is promoted in a rich learning environment which provides a safe space for babies to explore and includes familiar elements of home, resources that support babies' physical movement and collections of objects that encourage multi-sensory exploration, such as 'treasure baskets' and heuristic play.

As they get older, babies enjoy having containers to empty and fill, malleable materials to mould, water and dough, paint play, things to build with and things to knock down. Through these experiences they begin to develop an understanding of shape, space, and number. They also gain confidence in their ability to control their own learning.

Harry (aged 4 years 5 months) came into the nursery one morning shouting, 'I have a *thousand* ideas'. His nursery encourages children's creative thinking and supports children in developing their ideas. It is providing an enabling environment where children can confidently embrace new challenges. The relationship that Harry has with his key person is an important aspect of this trust.

For this reason, it is vital that practitioners:

- share positive beliefs about young children learning mathematics
- are aware of the mathematics that arises through children's self-initiated play
- have high expectations of young children's mathematical understanding
- understand babies' and young children's mathematical development, learning from reflecting on observations and through discussions with their team, and use this knowledge to 'tune into' the mathematics that children explore within their play. (See also *Development Matters,* EYFS, 2007a.)

Parents

It is important for practitioners to discuss and share insights with parents about their children's interests and play, both at home and in their early childhood setting. This collaborative dialogue can only enrich understanding and will provide a rounded view of children's developing mathematical knowledge and successes. (See EYFS Principles into Practice, Card 2.2.)

Extending children's mathematical learning

Observing children's play will help practitioners to value their growing mathematical understanding and reveal ways to support this development. Children's interests are powerful catalysts for mathematical enquiry and will provide a strong starting point to support and extend their mathematical thinking. Opportunities for problem solving, reasoning, critical thinking and reflection are vital if children are to make the most of their emergent understanding of mathematics.

Sensitive adults who value children's ideas and support children's play and mathematical explorations through collaborative dialogue help to 'scaffold' children's thinking. Practitioners can help children go beyond what they already understand and can do. Thinking, making meanings and understanding are significant aspects of mathematics.

Engaging in discussion with children means that the adult is genuinely interested in learning *from* the children about their ideas. Using open questions encourages children to talk about their thinking. This will allow ideas to be co-constructed and shared, and meanings to be negotiated and understood.

For further information on young children's talk, see: *Mathematical Vocabulary*, 2000 and 'talking hotspots'.

The 'talking hotspots' activity helps practitioners to evaluate how the environment supports the speech, language and communication development of the children and identifies places where authentic conversations between children and adults can take place. This will include discussions about mathematical issues, for example: the cost of items in the shop, the time a train will arrive and how late it could be, how long the ribbon needs to be to wrap the present, how many cakes are needed at the party, how much taller one sunflower is than another, the patterns on pairs of socks. (See 'talking hotspots' from *Every Child a Talker: Guidance for Early Language Lead Practitioners,* 2008, page 15.)

Food experiences help children explore quantities and space, shape and measures. These children are fascinated by the movement of the whisk as it goes round and round.

Making spaces that children can go inside, up and through helps them to explore spaces with their whole bodies. This also gives them opportunities to talk about positional language in an authentic way.

Placing pebbles in a basket outside with different sized containers and calibrated balance scales can support children's expectations of quantity and size.

These children are really interested in building as they have watched the real builders outside add an extension to their nursery. The nursery provides spirit levels and other building equipment inside and outside the nursery to further this enquiry.

The children have made a shelter and have had to search for sticks and branches of the same and different sizes. Now they have an exciting space to play in.

In this under-threes environment a space has been set up for the children to crawl in and go up – vitally important for awareness of space, direction and distance.

Ross has built three blocks on top of one another and has decided to measure them, using a paper tape measure.

Aimie is using the raffle tickets as birthday invitations for her friends. She says the numbers she knows as she gives them one each.

In this Reception class the teacher has used a display to draw the children's attention to numbers around them.

Number lines everywhere stimulate discussion about numbers and the children use these within their play.

Lauren has found a clipboard on which an adult had been marking children's choices for snack time. She has decided to help, making green marks over what the adult had written as she asks her friends what they would like.

Large calculators provide use of technology and interaction with numbers.

Providing challenges in the outdoor area stimulates talk about height: 'I am going to jump off the highest one.'

Numbers on the stairs encourage counting up and down in a sequence when climbing.

Young children love to have their own noticeboards where they can add their own graphics..

Creating a rich play environment to support PSRN

Mathematics does not depend on specific mathematical resources but on children having opportunities to develop mathematical concepts and understanding. When you have read the other chapters in the booklet you might find it useful to carry out the activities and audit provided in the Professional Development Meeting on pages 42–50 of *Early Years Quality Improvement Support Programme* (DCSF, 2008) entitled 'Developing a high quality learning environment, indoors and outdoors, which supports PSRN'. This can be found at **www.standards.dcsf.gov.uk/nationalstrategies**.

00861-2009BKT-EN

3. Children's mathematical graphics

This chapter builds on the DCSF publication *Mark Making Matters* (DCSF, 2008b), which emphasises that opportunities for children's early graphics should be threaded through every area of learning.

What are children's mathematical graphics?

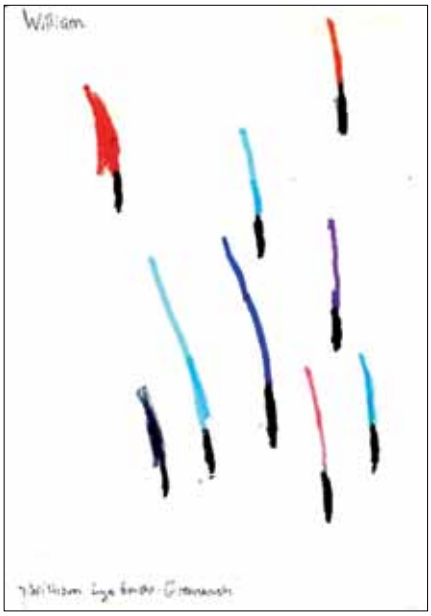
The term *children's mathematical graphics* was originated by Carruthers and Worthington (2003). It is used to describe children's own marks and representations that they use to explore and communicate their mathematical thinking. Research into *children's mathematical graphics*, (Carruthers and Worthington, 2006) has revealed young children's development of their early mathematical thinking as they explore the symbolic 'written' language of mathematics. These graphics include: scribble-marks, drawings, writing, tally-type marks, and invented and standard symbols including numerals. Young children's graphical exploration 'builds on what they already know about marks and symbols and lays the foundations for understanding mathematical symbols and later use of standard forms of written mathematics', Carruthers and Worthington (2006).

The EYFS PSRN emphasises that practitioners should:

'Value children's own graphic and practical explorations of problem solving' and observe 'the context in which young children use their own graphics.'

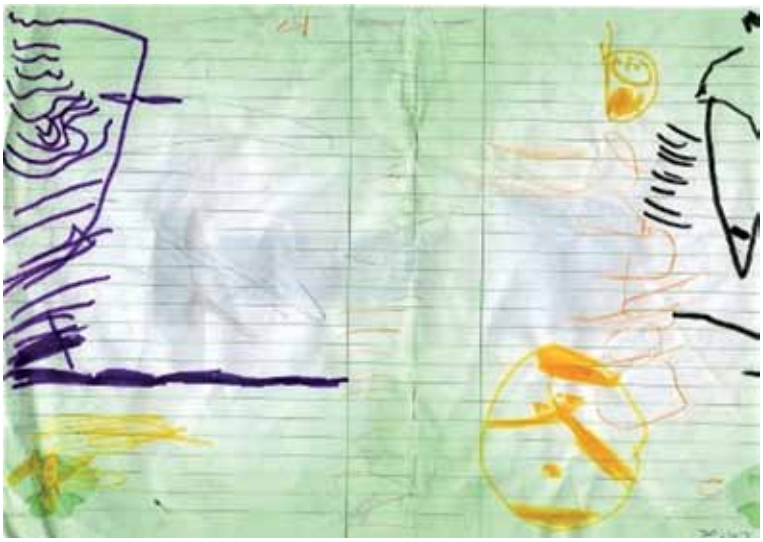
What do children's mathematical graphics look like?

Children's mathematical graphics support their developing mathematical thinking, and the following three examples illustrate part of this continuous process, making visible the sophistication of children's creative approaches to problem solving, reasoning and numeracy. The first two examples show children representing their mathematical thinking as they count. The third example shows how Kamrin, an older child, still uses his own more-complex graphics to help him reflect and work on a mathematical problem concerning division.



William's 'light sabres'

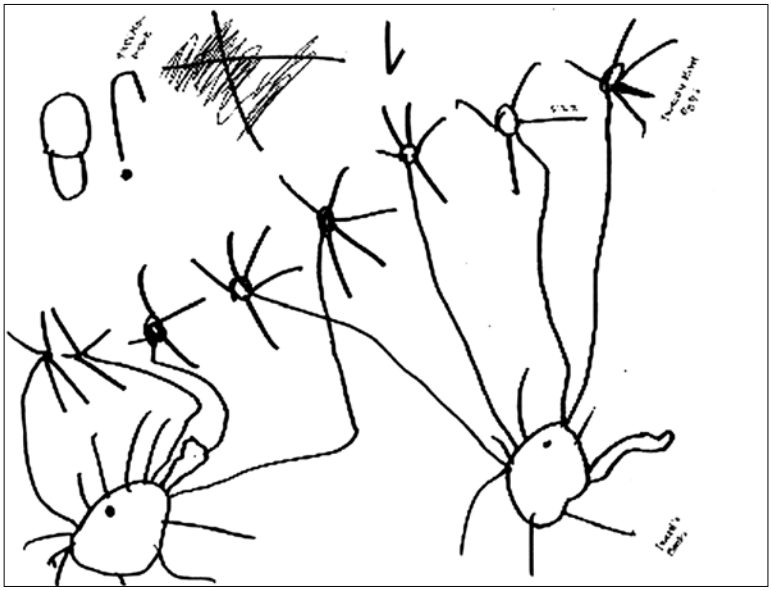
William (4 years 6 months) is fascinated by *Star Wars* and often plays out various scenes with his friend. He has drawn a light sabre for each of his favourite characters from the *Star Wars* film. The colours are important; for example, red is for 'Darth Maul'. His teacher explained, 'William draws hundreds of these pictures at the moment – either of light sabres or bows and arrows from the *Lord of the Rings* – they seem to be linked to his fascination with counting or amounts and he will count all sorts of objects, again and again'.



Chloë's register

Five-year-old Chloë has brought her own exercise book from home and says she is 'making register'. She has written the children's names down (the squiggles on the left) then counted them to see if she's made the right number of marks. She has counted to 4 and then counted random numbers to 20. She has used number strips to check and count how many children were in the class and decided to use tally-type marks to represent the number of children who were away. Chloë has a severve speech and language

disorder. Her own choice to make a register encouraged her to communicate her ideas in an alternative way. Her teacher was delighted with this, and also that Chloë wanted to explain what she had done.



Kamrin – 'Can 8 be shared equally between 2?'

Kamrin (5 years 7 months) invented his own way of checking if his chosen number, 8, could be shared equally between two. In this example he wrote '8?' and then added a cross to show that he thought it could not. On reflection he invented 'tweedle birds', giving each in turn a tweedle bird egg until he had shared the 8 eggs. Finally he scribbled over the cross he'd first written and added a tick to confirm that 8 could be shared equally.

Why are children's mathematical graphics important?

Young children need opportunities to develop their own understanding of the abstract written language of mathematics. The EYFS emphasises the importance of 'building on what young children already know and understand' and this is especially important for 'written' mathematics.

Through their own *mathematical graphics*, young children build on their understanding of how they can use their marks, symbols and drawings to represent their thinking. In a sense they are representing their mental mathematics on paper. Children's explorations with their *mathematical graphics* in play also underpin and support their 'written' calculations. Calculation is a complex undertaking and counting lays the foundations of this process. Children's understanding of written mathematics flourishes if they are encouraged to represent their own mathematical thinking when they are unable to work something out mentally. This also helps them work out their own strategies for solving problems.

Practitioners need to share *children's mathematical graphics* with parents and support them to develop an understanding of the important part that they play in children's mathematical development.

What's the difference between 'recording mathematics' and 'children's mathematical graphics'?

Understanding *children's mathematical graphics* is to recognise the difference between children *recording* a piece of mathematics after they have already done the maths, as opposed to children *representing* their own mathematical thinking in contexts that are relevant and meaningful to them.

When children are asked to simply *record* mathematics after they have worked something out (perhaps using practical resources such as little plastic bears), there is little scope for supporting or extending their mathematical thinking. In contrast, when children are encouraged to *represent their mathematical thinking* as they go along in contexts that are meaningful to them, they are able to use higher-level thinking skills (see Carruthers and Worthington, 2008).

When mathematical experiences are rooted in children's individual interests and fascinations, it increases their engagement, motivation, and desire to learn. This deepens their understanding of the abstract written language of mathematics, including calculations, and supports all aspects of problem solving. By having opportunities to represent their thinking as they explore and make meanings through their play, children's confidence in themselves as mathematical problem solvers and thinkers flourishes.

Key attributes of *children's mathematical graphics* include:

- children making their *own* choice to represent their mathematical thinking
- children making their *own* meanings – both independent and co-constructed (rather than copied)
- children's *own* graphics (drawings, writing, symbols, marks)
- children's *own* layouts
- children challenging themselves as they explore their mathematical thinking and meanings, communicate their ideas and solve problems.

> 'For children to become (young) mathematicians requires creative thinking, an element of risk-taking, imagination and invention – dispositions that are impossible to develop within the confines of a work-sheet or teacher-led written mathematics.'
>
> **Carruthers, E. and Worthington, M. (2003) 'Research uncovers Children's creative mathematical thinking, Primary Mathematics, Vol. 7/3 (Autumn).**

Why is play so important?

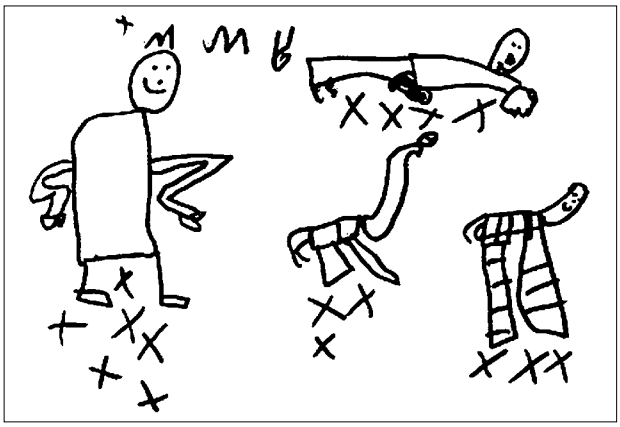
Children's mathematical graphics begin in their imaginative play, as they explore, make and communicate their personal meanings. This is often referred to as 'symbolic play' since children use actions, speech or resources (for example, junk materials) as symbols or signs to mean something specific. For example, in role-play Jemima pretended that a flat stone with grass and gravel was a plate with 'dinner'.

Often, between the ages of 3 and 4 years, children begin to attach mathematical meanings to some of their marks and representations, using their marks as symbols to think about quantities and numerals.

Children use their own symbols in flexible ways: This helps them understand that written symbols can be used to carry different meanings for different purposes. The following examples show children's flexible use of signs (crosses).

Jazper (4 years 0 months) has used a cross to represent an aeroplane (drawing).

Mark (3 years 9 months) writes these crosses to say 'No! Keep out!' (as 'writing').



Tommy uses crosses to show which animal each of his friends liked best (mathematics: collecting data).

Tommy's class had been on a visit to the zoo and were arguing about which the 'best' animal was. Tommy (4 years 7 months) decided to ask his friends, and after he'd drawn a lion, a crocodile, a giraffe and a zebra, went with his clipboard to collect his data. The choice of crosses was his idea.

Keeping scores

Several Reception class children had chosen to play ball, taking turns to throw it into a net. One child decided to keep a count of his scores on paper on a nearby easel, and wrote his name to show that he had scored one goal. Others joined in, representing their scores by writing the quantity they had, such as '3', '4', '5' or '1'. Some wrote tallies and Ellie wrote her name twice, to signify 'two goals scored'. Rakeem (5 years 1 month) had his own ideas and drew a Christmas tree in a pot with a circle around it, to show the single goal he had just scored. Jody (4 years 11 months) was intrigued by Rakeem's idea and when she scored two goals Jody decided to also draw a Christmas tree but this time, drew two pots beneath it as her personal sign for '2 goals'.

'The foundations of early written mathematics begin in children's make-believe play, for it is here that children begin to understand the relationship between meanings and symbols.'

Carruthers, E. and Worthington, M. 'Children's mathematical graphics: young children calculating for meaning' in Thompson, I. (Ed.) (2008) *Teaching and learning early number.*

Supporting children's mathematical graphics

The recent publication *Progress Matters* emphasises that 'children should be supported to pursue their unique paths in their learning and development' (DCSF, 2009b). *Children's mathematical graphics* are diverse, creative and individual, and if practitioners are to support and extend children's mathematical thinking they first need to understand it, tuning into their play, observing, listening and, if appropriate, joining in.

Practitioners will need to review the learning environment, inside and out, to ensure that children are experiencing rich, play-based contexts for problem solving, reasoning and numeracy. A range of mark-making resources should be freely available so that children can make choices about how they represent their thinking, depending on the mathematics in which they are involved at the time.

Practical pedagogical points to support children's mathematical graphics

- Tune into and value children's own meanings.
- Value *children's mathematical graphics*.
- Discuss the *children's mathematical graphics* with them to help them reflect on the meanings of their own and other children's graphics.
- Observe closely and annotate *children's mathematical graphics*; this will allow adults to uncover children's thinking and meanings sensitively.
- Model *mathematical graphics* indirectly – it is important for children to see different ways to represent mathematical thinking.
- Create a culture in which children use graphical media independently throughout the environment and make a wide variety of papers, pens, pencils and other mark-making equipment, easily accessible in all areas of provision, indoors and outdoors.

4. Numbers as labels and for counting

This strand of EYFS PSRN is about how children gradually know and use numbers and counting in play, and eventually recognise and use numbers reliably to develop mathematical ideas and to solve problems.

Children's understanding of number starts from birth and develops gradually. You will find useful information about development in EYFS PSRN *Development Matters*.

Babies as young as five months appear to be aware of quantities, and they can notice changes in amounts of objects. Providing babies with objects to explore supports this early development of number.

As young as eight months old, children are developing an awareness of number names, and include these in their speech, as soon as they begin to talk. As children listen to the talk around them, they are introduced to number through opportunities that occur in everyday life, and experience a variety of number rhymes. This supports their growing knowledge of number names.

00861-2009BKT-EN

It is important to remember that when children learning English as an additional language (EAL) are developing understanding in counting they will have knowledge of numbers in their own language from home and it is important to acknowledge this as well as gradually introduce them to the English number names.

Number rhymes are a wonderful and joyful way to support children's language, and nursery rhymes are historically one way that parents introduce their children to mathematical language. Number and counting rhymes are used in most settings. They are important for the rhythm and rhyme that supports all language development but are also important for developing young children's awareness of all the skills involved in counting if they are used appropriately. Children's home culture may also nurture their own favourite rhymes, and so rhymes in children's home languages should be supported and encouraged.

How do children develop counting skills?

When you observe very young children you will see that they start to count spontaneously and later they begin to refine their skills by pointing their finger at the objects they are counting. They often try to get all the names of the numbers they know in their count as they pass their finger along the objects. They also reuse numbers. If they have not finished and they have used up all their known numbers, they will begin using the same numbers again. For example, Sophia (2 years 7 months) decided to count the eight shells she had collected at the beach. She lined them up carefully and then tagged numbers to them pointing as she slid her finger along the shells, quickly counting out loud: '1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5.'

In their drive to make meaning, children are eager to experiment as they acquire new small bits of mathematical knowledge, and it is extremely important to respect their developing understanding and not expect or wait for 'perfect' counting sequences or the right answer. Children's learning of numbers is holistic, and they do not learn numbers in the traditional order, for example they do not learn to count 1 then 2 then 3, etc. Larger numbers fascinate some children (often boys); they often talk about '100' or '1000' and know that it is a 'big' number.

By valuing children's partial understanding, children will develop enthusiasm for number and become confident mathematicians. It is important to acknowledge all children's counting and to be aware of their developing competence.

Carefully select number rhymes to include those that children are familiar with from home.

Make sure the rhymes include:

- counting back and counting forward
- 'no' or 'none' (Five little ducks went swimming one day)
- counting in pairs (2 ,4, 6, 8, Mary at the cottage gate)
- counting to five, ten and beyond.

Practitioners should plan to introduce rhymes systematically, so that children can increase their repertoire of favourite rhymes.

Ensure children have opportunities to revisit rhymes independently – through small-world play, with puppets, soft toys and other props, including magnetic props.

Consider the repertoire throughout the year to ensure the rhymes remain interesting and challenging.

As counting skills develop further, children begin to understand:

- **one-to-one correspondence** – when a child points to each object individually and they count and match a tag (a number) to each object they are counting
- **the same order** – you will observe children gradually working towards the knowledge that the number words must always be said in the same order
- **anything can be counted** – children, if given an assortment of opportunities to count in a variety of ways, for a variety of purposes, will develop the understanding that anything can be counted not just the objects in front of them. For example, Muna (4 years 2 months) said that he had five steps in his house. The steps were not there in front of him but he could count them in his mind. Jasper (3 years 9 months) counted the music chimes he heard – '7 chimes', he said
- **it does not matter which object you start with when counting** – children will understand that you can count objects in any order and you will get the same result provided that you count each of them once and use the counting sequence.

Key concepts in the development of children's counting

- *I know that the last number I say in the count is the name used to represent the size of the group.*

  When asked the question 'How many?' Jake (4 years 1 month) would repeat all of the numbers in the count. For example, 'Jake, I wonder how many beetles are in the jar?' Jake counted '1, 2, 3, 4'. The practitioner asked again, 'So how many are there in the jar?' Jake replied, '1, 2, 3, 4'.

  Five months later Jake (now 4 years 6 months old) carefully counted seven spiders. When we asked Jake how many spiders he had, he said, '7'. Jake now knows the last number in the count is the value of the whole set and is able to express that.

- *I don't need to count; I can see that there are three there.*

  Once children start counting fluently they recognise small quantities up to three without counting. Subitising is the term used when children can judge the number of objects in a group rapidly, accurately and confidently without counting them and then count on from the group subitised. Many children subitise when looking at dice and patterns, and we can capitalise on this. For example, Joshua (5 years 5 months) counted the leaves he collected from the forest. He put the leaves on the table and could immediately see a group of three leaves within the larger amount. He began by counting at three and continued counting all the other leaves '4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14'. It is important to acknowledge and develop this skill. When children are really fluent counters then it is not always necessary to start the count at one.

- *I know I need to check my counting because sometimes I miscount.*

  When children self-check their count they have moved away from the preliminary counting focus (i.e. the skills) and moved on to greater accuracy. This is a vital development and shows clear understanding of the counting procedure. It is important to note that some children obtain this concept themselves. Other children are helped greatly by the practitioner modelling or demonstrating this process in an appropriate context. However, children need many experiences of counting before this kind of input will be effective.

Numbers as labels

Numbers are not always experienced in the context of counting. 'Numbers as labels' are written numbers that are used to label things such as door numbers, numbers on birthday cards, numbers on clocks and train numbers.

Children are fascinated with the numbers of their lives; these numbers have personal meaning to them. Their age number is important to them, just like their name. Their age becomes part of their identity, for example Yasmin (aged 3 years 4 months) pointed to the number three, saying 'That's me. I am three'. Many children will be interested in the ages of the people in their families, their car numbers, their telephone numbers and the number on their front door. Any opportunities to support and extend their thinking on these kinds of numbers will enhance the children's development.

What about number tracks and lines?

It is essential to display number lines because children need to have a reference for their conversations. Adults can easily refer to the number lines to support any number conversation or group activity. Often children will look at their age number or door number to initiate a conversation. Number lines of different lengths are useful to show children that numbers go beyond 10 and that the children can also see the number sequence and how numerals are written. Numbers that can be easily moved on a string are excellent for using flexibly where children can discuss, for example, the numbers they want to hang on the line.

Number tracks are also useful because children can jump on them or put objects on them. Number tracks are often drawn outside for the children to play on. Adults can initiate games, for example, 'I am on number 3. I am going to jump to number 7.'

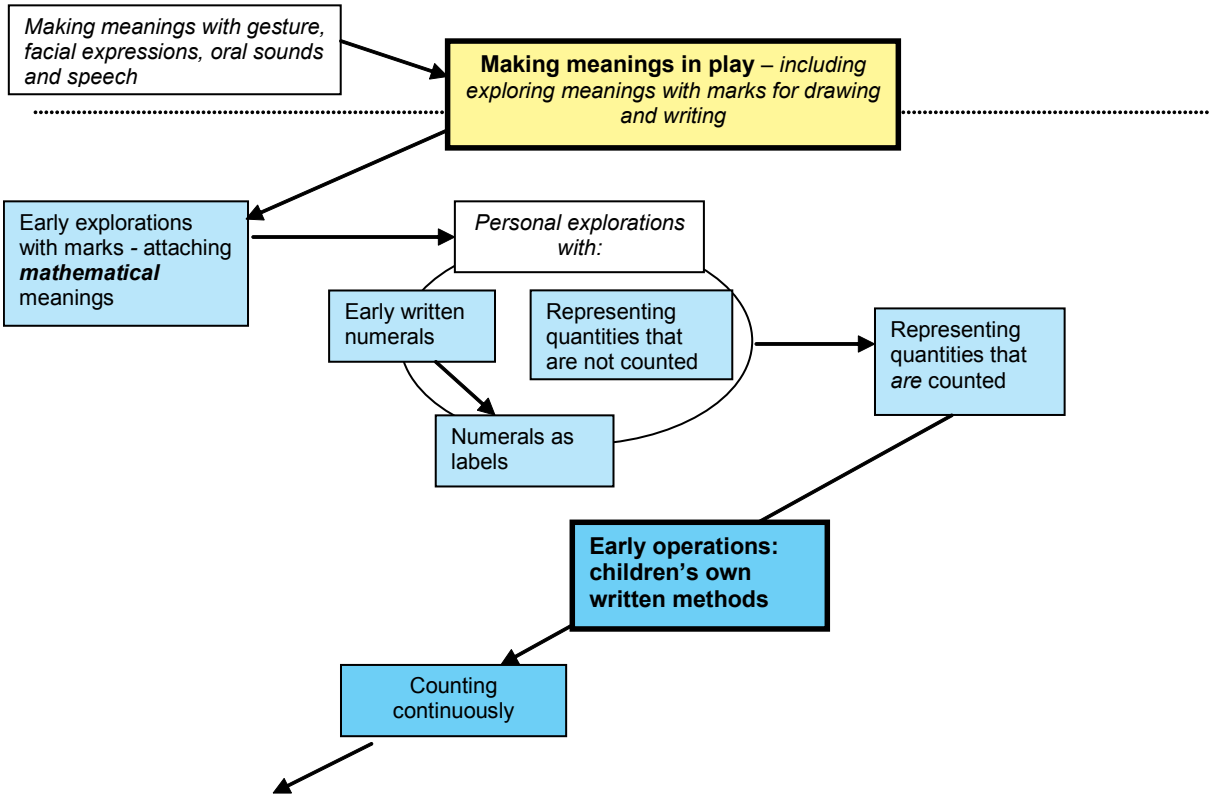
Writing numerals

Children develop their knowledge of number through language and practical experiences.

Chapter 3 of this booklet considers how young children also express their mathematical thinking through their own *mathematical graphics*. Children will also begin to explore ways of writing numerals.

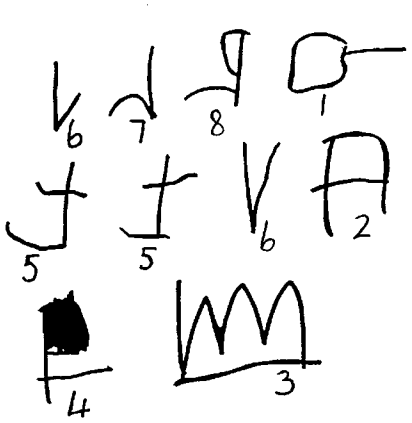
The taxonomy of children's mathematical graphics: beginnings in play

The taxonomy below shows young children's development of their mathematical thinking, explored through their *mathematical graphics* from their early marks to counting.



This first part of the taxonomy of *children's mathematical graphics* shows a range of examples from children's early exploration with marks to representing quantities. These very early *mathematical graphics* can be observed from children from two to five years old.

Early exploration with mathematical marks	Young children begin to differentiate between marks they use for drawings and those they use for 'writing'. They begin to ascribe *mathematical* meanings to the marks they make in play.
 	**Sam and the calculator** (3 years 6 months) Sam has been watching his friend Bradley play with a calculator and occasionally writing on a piece of paper as he presses the buttons. Sam wants to be a part of this play and when he has fetched a calculator for himself, the two boys talk about the numbers they press, often choosing their age number: '3'. Sam decides to make marks of his own on a piece of paper as they play.

Early written numerals	Children refer to some of their marks as numbers and begin to explore ways of writing numerals. Some of their personal symbols may include features of standard written numerals.
 	**Alex's numbers** (4 years 11 months) Alex loves writing and, on this occasion, chooses to explore numerals using his own symbols. He adapts the symbol '6' for '7' and '8' and uses elements of standard letters and of numerals he knows. He is consistent in repeating '5' and uses the first letter of his name to stand for '2'. Alex's explorations illustrate just how much he already knows about written symbols and number, showing that Alex knows his numbers. He soon comes to understand and use standard written numerals.

Numerals as labels	This is when children use numbers that they see in their environment as labels for particular purposes and contexts. Knowing and talking about numbers is different from writing them. When they choose to write numbers in context, they have converted what they have read and understood into standard numerals.
 	**Jessica's clocks** (4 years 6 months) Jessica wanted to write all the 12 numbers in the clock the first time but they did not quite fit so she has tried several times. She gives this to her key person explaining 'It's nearly milk time'. She is making connections with what she knew about time in the nursery (i.e. milk time) and clock time, and finally carries her sign around the nursery to show the other children.

Representing quantities that are not counted	Young children explore links between their early marks and meaning as a quantity in a general sense.
 	**Joe's spider** (3 years 9 months). Joe is exploring and playing with a set of toy spiders in the nursery. Joe decides to draw a spider and tells the nursery teacher, 'My spider is Hairy Maclary and he has eight legs.' Joe represents his idea of many legs in his drawing: he shows a growing awareness of number and quantity and is able to describe it.

Representing quantities that are counted	Children begin to: ● count the marks or items that they have represented ● represent items they have already counted. This development leads to the beginnings of early calculations and all other aspects of written mathematics that involve counting.
 © E. Carruthers & M. Worthington. Published by Sage, 2006	**Jenna's raindrops** (3 years 9 months). Jenna draws and counts raindrops in the graphics area. Perhaps the coloured pens remind her of raindrops, or this may be a current interest of hers since many children love to draw rainbows. Jenna finally counts each vertical column before proceeding to the next to reach the total she has drawn.

Practical pedagogical points to support the use of numbers as labels and for counting

- Remember that pedagogical approaches to Early Years mathematics usually centre on children's interests and their play. Opportunities for mathematical thinking will naturally arise from these experiences. However, if practitioners concentrate on bits of the process, for example insist on the child learning numbers 1 to 5 before they are allowed to explore 5 to 10, then the child's thinking will be restricted.
- Plan opportunities for mathematics in everyday routines.
- Provide opportunities for mathematical conversations, listening to the children and providing open questions.
- Audit your environment:
  - Do you have number tracks/lines of different lengths, in a variety of places and going beyond 10 and 20?
  - Are they displayed at child height?
  - Do you have mathematical equipment and a variety of resources for graphicacy to use throughout the setting?
- Provide a variety of opportunities for interesting counting experiences that have purpose and meaning for the children.
- Carefully select a variety of number rhymes including those that children are familiar with from home. Make sure they include rhymes that:
  - count back and count forward
  - support the concept of 'none'
  - include the language of addition, for example 'altogether'.

5. Calculating

This strand of EYFS PSRN is about how children develop an awareness of the relationship between numbers and amounts, know that numbers can be combined to be 'added together' and can be separated by 'taking away' and that two or more amounts can be compared.

As children's counting knowledge develops, they acquire skills that are the foundations of calculation. Children's development from counting into calculation is seamless and each child develops their understanding of calculation in different ways and at different times.

Children:

- compare groups or quantities using one-to-one correspondence to find the difference – early subtraction
- share objects equally by counting how many in each group – early division
- count groups of the same number of objects and add them together – early multiplication
- count back from a fixed number when taking away – subtraction
- count on from a fixed number when combining two groups of objects – addition
- Find one more or two fewer by counting on or back – early addition and subtraction.

Children's counting skills are supported by knowing the everyday language of mathematics so that they can talk about and explore their ideas.

Addition – when counting two groups: 'How many do we have *altogether*?'

Altogether is a key word to use in the teaching of early addition.

Subtraction – 'That's great, Charlie's made a cake with candles because it's Harry's birthday. There are six candles and Harry's four today – how many candles do you think we need to take away?' (at the dough table)

Knowing mathematical vocabulary is important for children to have a language tool for mathematics (see *Mathematical Vocabulary*, 2000 in references). Other important vocabulary for calculation includes 'as many as', 'more', 'most', 'less', 'least', 'make' and phrases like 'left over', 'the difference between', 'too many' and 'too few'.

Mathematical vocabulary needs to be set in purposeful contexts and not taught as a list to be remembered

Open questions should be used to encourage and support children's problem solving, reasoning and creative thinking in mathematics.

Children use their developing knowledge and understanding to solve mathematical problems

As children's increase their knowledge of calculation, they are also becoming proficient mathematical problem solvers. Babies and young children are naturally curious about their worlds and seek out solutions for problems they find within play and everyday experiences. Young children are constantly solving problems; they question, reason and persist in their own enquiries. Problems can be set by adults but are often initiated by the children themselves. When problems arise from contexts that are personally meaningful and relevant to the children, their curiosity is sparked and they are likely to be highly motivated.

Children informally explore their ideas about calculations as they combine amounts, share quantities and calculate totals within their self-initiated play.

For example, Sophie (2 years 9 months) is looking in the fridge and tells her mum, 'We have only five yogurt pots.' Counting them, she realises that there are six people: 'That makes one for daddy, one for mummy, one for Dervla, one for Melanie.' [pause] 'I can share with Nicola.'

Some of the problems that naturally occur in children's play provide contexts in which children can use their developing mathematical ideas and methods. It is important that children are challenged by mathematical problems and can choose their own methods. If practitioners are providing problems for children to solve, they should be set in a wide range of meaningful contexts and not be limited to problems involving direct calculations.

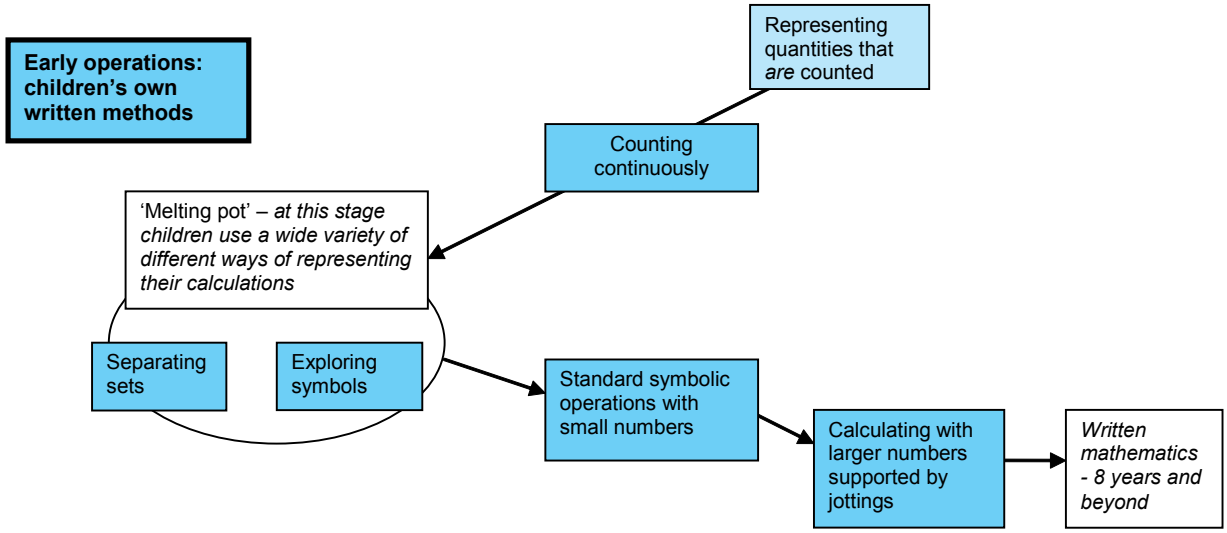
> 'If children already know or are told the method to use, then they are not problem solving.'
>
> **Gifford, S. (2005) Teaching Mathematics 3–5, Maidenhead: Open University Press/McGraw Hill**

Collaborative dialogue (*Supporting Young Children's Sustained Shared Thinking: An exploration*. Dowling, 2006) between adults and children will support their thinking and ideas, as they explore different ways in which mathematical problems can be solved. This is especially important when children use their own *mathematical graphics*, as shared talk will help them to think about the various ways in which they have used marks and symbols to support their calculations and problem solving.

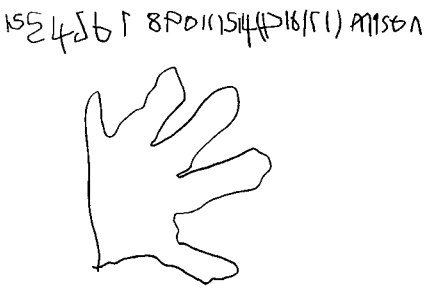
Children's shared talk about problem solving does not always lead to *mathematical graphics*; the use of language is valuable in its own right.

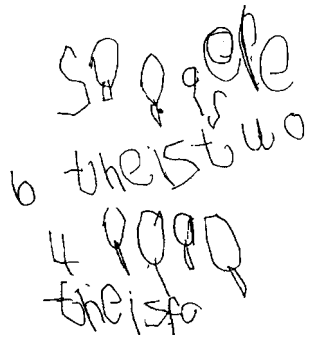
The development of children's own written methods for calculations: children's mathematical graphics

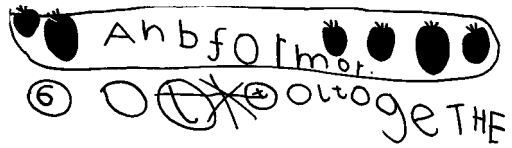
Continuing the journey into children's mathematical graphics

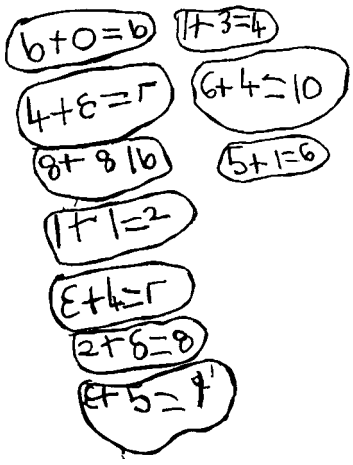


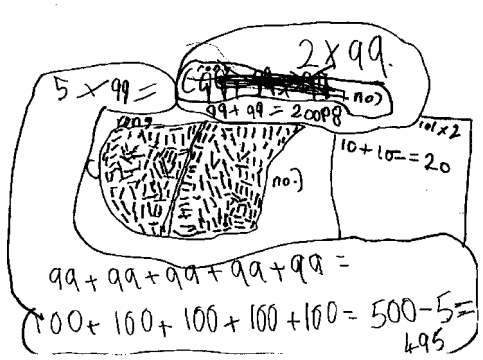
This part of the taxonomy shows a range of examples from children who are between four and eight years of age. They have used their own *mathematical graphics* to help them think about mathematical symbols and calculations and to solve mathematical problems.

Counting continuously	At this point children represent two sets of objects or numerals arranging them on paper in a continuous line. The two sets are not separated and items are counted continuously to arrive at a total (rather than added). They understand the need to count everything to arrive at a total.
 	**Alison's breakfast café** (5 years 1 month) Alison wants to work out the total number of children who will be coming to her class's 'breakfast café', counting the seven children and the teddy each child will bring. She represents them as a string of numerals, continuing to count on from the total number of children. Deciding to re-count (to check) she finds she had written too many numbers and puts brackets round '15, 16, and 17' but does not explain the hand that she has drawn: it may represent the action of 'adding', although we cannot be sure.

Separating sets	Children use a range of strategies to show that the two amounts that they are adding are separate, for example by: ● grouping the two sets of items to be added, perhaps by leaving a space between them or by representing them on opposite sides of the paper ● separating the sets with words ● putting a vertical line between them ● putting an arrow or personal symbol between the sets ● drawing rings (boxes) around each set.
 	**John – adding grapes** (5 years 5 months) John is adding two sets of grapes that he will later eat. He chooses to use words to help him think about his calculation, using drawings of grapes and words to separate the numerals. He reads this as '2 grapes, there is two; 4 grapes, there is four' and then writes the total on the left of the paper. The words he writes separate the numerals of his calculation.

Exploring symbols	Children explore the role and use of symbols by: • using personal or invented symbols, or approximations of the standard symbols • leaving a space between the sets to imply that a symbol (operant) is needed here. Often children who do this will read what they have written as though the missing symbol was there • combining drawings, words, numerals and personal symbols.
 	**Louisa's strawberries** (5 years, 1 month) Louisa has selected two small quantities of strawberries from a plate her teacher put on the table for her group. She puts them in two small bowls in front of her, and chooses to use paper to help her understand the symbols when 'writing' calculations. Louisa combines drawings, writing and numerals, drawing a box around the first part of her calculation and finally reads this as '2 strawberries and four more. Altogether 6.'

Standard symbolic operations with small numbers	This stage arises directly out of the preceding ones – all their previous knowledge combines to support simple calculations with small numbers. Children: • use standard numerals and symbols in their calculations in a horizontal layout • show the operation in three steps • sometimes separate calculations from each other by a line, circle or box.
 	**Anna's dice game** (5 years 7 months) In a mixed Reception/Year 1 class, Anna's teacher has introduced a game for the children to play in pairs, each pair with two dice. She invites the children to 'find out how many (dots) you get altogether – when you throw the two dice'. This game provides an opportunity for the children to explore the mathematics in different ways, as counting and perhaps as calculations. Anna (in Year 1) decides to use the opportunity to calculate, using horizontal 'sums' with standard symbols as she combines the totals.



Alison's 99 times table (7 years and 5 months)

Alison is trying to work out the 99 times table. She chooses to begin with '2 x 99', then after much crossing out puts down '99 + 99 = 20098'. This shows logical thinking, since this is how '298' sounds when spoken. (Although Alison's answer is incorrect, her thought process and experimentation is valid. She uses a range of strategies that show the processes she explores, generally self-correcting as she works. Her final strategy for 99 x 5 is to use repeated addition, rounding each '99' to '100' and then subtracting '5' from the combined total to arrive at her answer.

'Children should be encouraged to base their written methods on their own preferred mental calculation methods. They should be working in an environment that attaches value to idiosyncratic written methods, and should be praised for inventing their own notations or jottings.'

Thompson, I. (Ed.) (2008) What do young Children's Mathematical graphics tell us about the teaching of calculation? In Teaching and Learning Early Number, Maidenhead: Open University Press, p. 158, (2nd Ed.). Used with kind permission.

Some of the problems that naturally occur in children's play provide contexts in which children can use their developing mathematical ideas and methods. It's important that children are challenged by mathematical problems and can choose their own methods. For children in Reception, problems should be set in a wide range of contexts that have real meaning for them, and sometimes for real audiences (i.e. people who need to know): this will help children make personal sense of the mathematics.

Practical pedagogical points to support calculation

- Support children's play inside and outside with a mathematically challenging environment.
- Provide an environment where children can self-select the tools they want to use to aid their enquiries.
- Listen and observe children, to understand their mathematical thinking.
- Provide a culture of enquiry where children's own ideas are at the heart of planning.
- Share and discuss children's ideas of calculation strategies with them.
- Make regular times for small-group discussions about children's own personal mathematical problem solving.

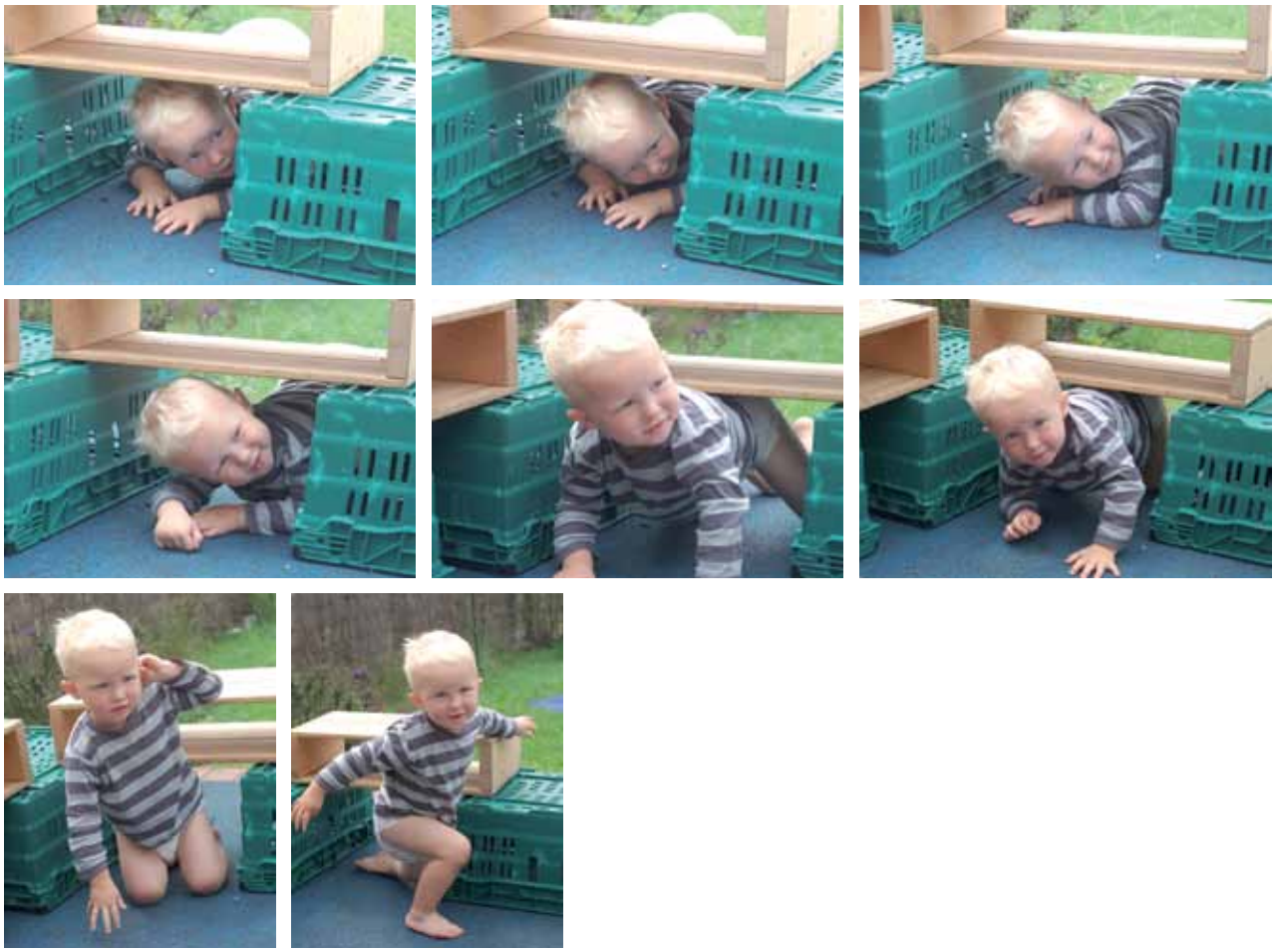
6. Shape, space and measures

This aspect of EYFS PSRN is about how, through talking about, exploring and manipulating shapes and quantities, and developing appropriate vocabulary, children use their knowledge to develop ideas and solve mathematical problems.

Shape and space

As soon as children are born they naturally start to explore their world through space and shape. They appear to have an inbuilt ability to investigate objects and their environmental surroundings. They do this at first through touch and exploring objects with their mouths. Babies and young children will explore the mathematical properties of 3-D and 2-D shapes and space, position, movement and direction through a personal exploration of their environments. These first-hand explorations allow them to use their senses, playing with natural and everyday resources in their home and Early Years settings.

00861-2009BKT-EN



The shapes and spaces of the child's world increasingly open up as they begin to crawl and move in many different ways. Tunnels or arches, small spaces, dens and boxes all offer particular interest and enjoyment. Settings therefore need to provide a wide range of opportunities both inside and outside for children to explore space on different levels and in different ways.

00861-2009BKT-EN

Making beds and building dens

Kissy, Ross and Melis choosing the right-sized material to make a tent.

The three children have crawled into the very small space. They have tried to get as many people as they could into the space using mathematical language such as 'big', 'too big', 'small', 'little', 'beside', 'on top of'.

Spontaneous play offers valuable opportunities to explore this area of mathematics in personally meaningful ways. Different materials and resources including bricks and blocks; dens, role-play and small world play; train layouts; sand and water; play outdoors and dance all provide rich opportunities for children to explore shape and space, position, movement and direction. Junk modelling and free play with construction sets and malleable material such as clay also provide invaluable contexts for young children to investigate**.** In her important research into block play, Gura describes children exploring pattern through bricks and blocks, where 'the exploration and creation of shape and pattern took off in a big way' (Gura, 1992).

Use the *EYFS PSRN Effective Practice Guidance* to help plan appropriate opportunities for all ages of children.

Small cylinders – Joshua playing at viewing everybody through a cylinder.

Everyday language of shape and space

A child's use of everyday mathematical language should be accepted and valued. Adults can gradually introduce the mathematical language of shape and space, position, movement, direction, pattern, symmetry and measurement in contexts that are meaningful to the child. In the EYFS, it is important that children use language that makes sense to them and that standard mathematical language is introduced develop the meaning for children.

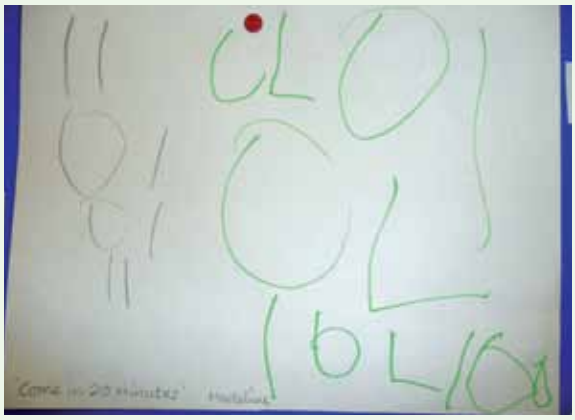
The naming of shapes can be less important than this hands-on exploration and should not be introduced to in isolation as a discrete activity, for example by holding up shapes and naming them at group times.

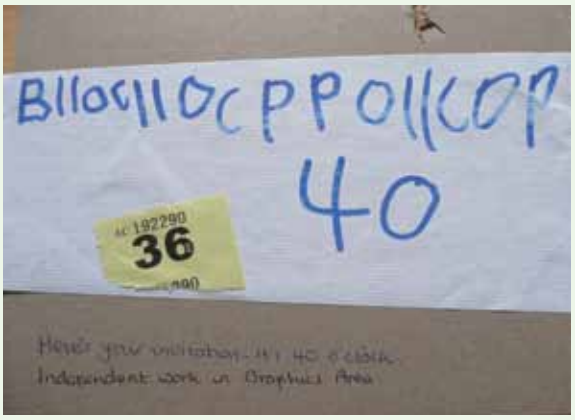
Measures: time

Babies and young children develop intuitive understanding of time through the rhythms of day and night, the duration of time, of time passing and speed of actions and movement (for example, quick/slow). Babies and young children explore many aspects of measurement through their patterns of behaviour, for example through lining things up, going round a boundary and filling containers.

In Early Years settings daily routines, order, predicting what comes next and recalling what they have done earlier during the natural course of events will help young children become increasingly aware of time. Talking about activities in natural contexts such as the time left before snack-time; growing seeds; discussing how long some cakes need in the oven; waiting for eggs to hatch; discussing how long before it is time to go home or how many bedtimes until a child's birthday the following week will all help link concepts of time and its language. Play that involves movement, such as walking and running; dance, movement and music (singing and percussion); picture story books relating to time and age and play with computerised toys such as the 'Beebot' also provided valuable experiences in this area.

Case study: writing invitations – nursery





Madeline and Zachary

Zachary had been talking about his birthday – his mum had said he could invite all the children in his nursery class to his party. This caused much excitement; most of the children had been to birthday parties and understood the role of invitations and there was a lot of discussion about which day the party was to be held on and what time the children should arrive.

Several children decided to write their own invitations! Madeline (3 years 5 months) used written symbols she had already noticed – vertical lines and circular symbols – explaining 'come in 20 minutes'. Zachary used a range of letter-like symbols and while he was thinking about numbers, also stuck on a raffle ticket. He wrote the '4' of his age, reading 'Here's your invitation. It's 4 o'clock.'

Zachary and Madeline were exploring their ideas of time through invitations. Knowing that the time was important, Madeline described her thinking by saying 'come in 20 minutes' and connecting that with making her own symbols. Zachary's focus was more on the clock time and he put his own number 4 down saying 'it's 4 o'clock.'

Measuring and comparing lengths

Provide children with a range of interesting materials and different lengths of ribbon and string, including real measuring devices such as a measuring tape, to explore. Observe the children and find out what they are interested in; for example some children may be interested in filling and containing (investigating ideas of capacity and volume). By providing different-sized containers in the outside and inside areas, children will be enabled to seize the opportunity to explore this area of mathematics.

Comparing lengths – Rosa (2 years 6 months)

Measuring heights and writing their own number

Other children may be interested in measuring heights. If they are given, for example, height charts or a tape measure, children will be able to use these materials to answer their own questions and pose other questions to their key worker.

Height charts should include standard and non-standard measures, such as handprints, and give children opportunities to write their own numbers.

Using measures

Cooking (investigating food) is a wonderful medium to develop children's purposeful use of measures. Providing a variety of equipment that children can use to measure and explore enables the children to solve their own measuring problems.

Thomas remembers that they used a timer last week to take turns. Pradesh finds two timers and the boys decide on a way of measuring time with two timers.

Taylor carefully divides the mixture equally into the eight-part cake tin.

Brandon decides to weigh the vegetables for his vegetable soup.

Practical pedagogical points to support exploration of shape, space and measures

- Observe and value children's explorations of shape, space and measures in their self-initiated play.
- Provide open-ended resources such as cardboard boxes, cylinders and other containers; tapes, glue and string and other 'junk modelling' materials (many of these resources can be easily found at local 'Scrap Stores' at very little cost).
- Audit your outside area to ensure that it provides a variety of opportunities to experience 'going through', 'on top of', 'under', 'over' 'going round' and 'between'. Children learn positional language best in purposeful contexts.
- Support babies and young children in exploring the properties of shapes. The naming of shapes should not be a focus.
- Include real measuring equipment such as measuring tapes and calibrated measuring jugs; analogue and digital clocks; timers, etc. in the play environment. For example, children could use kitchen timers to help them organise turn taking.

7. Transitions

Effective practice

As children move through the EYFS and into Key Stage 1, they will inevitably experience a range of transitions. *EYFS Principles into Practice* Card 3.2 tells us that settings 'should communicate and work together for the benefit of the children, so there can be continuity in their learning'.

Practitioners will need to plan together to ensure that children's early fascinations with mathematical ideas are celebrated and built upon, to extend their understanding and delight in PSRN and to maintain and strengthen their confidence in mathematical thinking. When children's experiences of change are gentle, sensitively supported by knowledgeable adults, they are more likely to develop a confident and positive mindset that enables them to embrace comfortably new challenges and adapt to new situations.

Building on what children already know and understand

'Discontinuity can arise when teaching does not take sufficient account of what children already know. Continuity of learning is supported when practitioners acknowledge the role of the earliest aspects of learning in later mathematical learning.'

Pound, L. (2006) Supporting Mathematical Development in the Early Years, p. 98 © Open University Press. Used with kind permission.

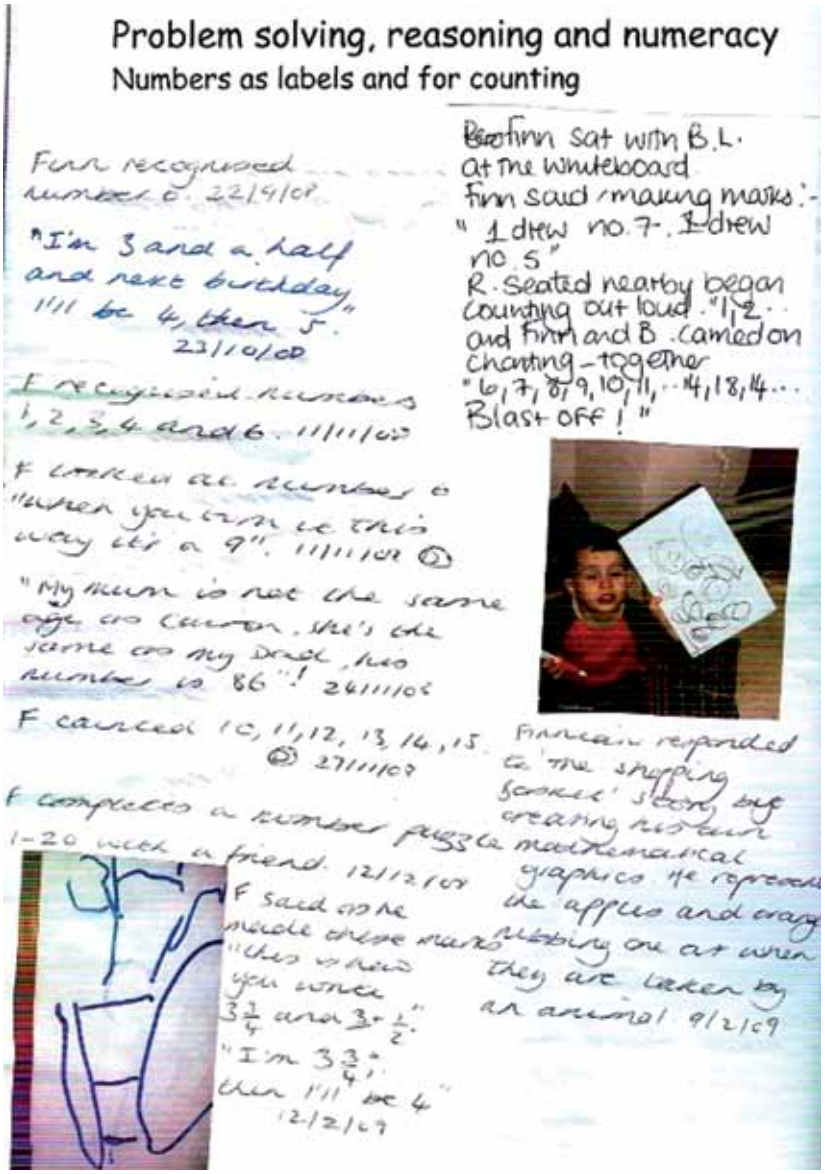
Practitioners, parents and childminders will need to prioritise time to share observations, talk together and, as equal partners, discover more about each child's developing mathematical journey to ensure continuity between home, setting and school.

Leaders and managers of settings and schools will need to find ways of enabling practitioners to work collaboratively with staff in both the settings from which their children have come, and also with staff in settings and classes to which their children will move.

In order to do this, practitioners should ensure that they have completed observations on children's developing PSRN and noted their progress. Many settings keep a learning diary for each child which supports staff and parent knowledge of children's development and learning, and provides a real child focus for staff discussion.

Parents should be involved in compiling any learning diary; for example, the first learning story in the diary could be a story from home about the child that the parents or carers have shared with their key person. Parents can then be encouraged to continue adding to the diary. Practitioners find that families are proud of the learning diaries and they provide a wonderful story of their child's time at nursery to show to their Reception class teacher.

Example of learning diary



A page from Finnian's learning diary.

2 years 6 months	Finn stands against the height chart: 'Look, I getting bigger.' 
2 years 8 months	Finn sings to himself at the lunch table: '3 little fingers, 3 little fingers, 3 little fingers tap, tap, tap, 2 little fingers, 2 little fingers tap, tap, tap.'
3 years 0 months	Pointing to the number line: 'that's 3 for Finn, 5 for Ciaran and 4 for Saoirfe.' • ***Starting to recognise numerals***
3 years 7 months	Finn recognised numbers '1', '2', '3', '4' and '6'.
3 years 8 months	Finn looked at number 6 and said 'When you turn it this way it's a 9'.
3 years 8 months	'My mum is not the same as Cairan, she's the same age as my dad – his number is 86.'
3 years 9 months	Finn sat with Brandon at the whiteboard, making marks: 'I drew number 7. I drew number 5.' R. sat nearby and began counting out loud, '1, 2…' and Finn and B. carried on counting together '6, 7, 8, 9, 10, 11 … 14, 18, 14… Blast off!' • ***Ordering numbers in sequence to '11'***
3 years 10 months	Finn responded to 'The Shopping Basket' story by creating his own *mathematical graphics*. He represented the apples and oranges, rubbing one out when they were taken by an animal. • ***Subtraction*** • ***Using mathematical graphics for calculating*** 
3 years 11 months	Finn drew his own watch 'Spiderman' after exploring a variety of clocks and watches at 'Talk Time'. Finn included some numbers: '3, 4 and 7'. With help from Brandon he cut his watch out and wore it for the afternoon.  • ***Interested in numerals as labels***

3 years 11 months	Finn completed a number puzzle with numbers 1–20, with a friend.
4 years	Finn said as he made these marks: 'This is how you write 3¾ and 3½. I'm 3¾, then I'll be 4!' • ***Interested in fractions***
4 years 1 month	Finn looked at the numbers up to 100 on the door: 'A hundred is like ten but with another zero, so that means it's bigger.' • ***Making deductions and explaining a theory*** • ***Exploring larger numbers***

Children's understanding of mathematics needs to be valued as they move from one phase to another so the practitioners and teachers can build on what children already know and understand. The Primary Framework will support the transitions from EYFS to Key Stage 1.

Practitioners should reflect on and review children's achievements

This should be happening throughout the EYFS. Practitioners should be considering all aspects of children's mathematical achievements.

The Williams mathematics review challenges practitioners to think more deeply about the EYFSP PSRN scale point 8: 'Relatively few children attain point 8, *uses developing mathematical ideas and methods to solve practical problems,* in any of the three mathematical assessment scales.' It recommends: 'It is essential that the Foundation Stage Profile (FSP) is analysed at scale point level, rather than simply looking at total scores' (DCSF, 2008a).

Problem solving, as mentioned in previous chapters, relates to children's language, thinking and explorations in all areas of PSRN. Children making decisions, adjustments and choices while making boats in the block-play area or exploring graphical symbols for their age as Finnian did, are examples of problem solving. Practitioners will be more confident to make judgements about scale point 8 when they feel more confident in their understanding of mathematics as a whole.

Children's search for meaning needs to be recognised and valued throughout the EYFS and Key Stage 1, with practitioners continuing to encourage children to build on their own *mathematical graphics*. At each stage of their developmental journey, children need opportunities to explore what they already know and understand about how marks, symbols, letters and drawings can all be used to communicate – and to use their own graphics for real purposes in meaningful contexts.

Moving from EYFS to Key Stage 1

In Reception and Key Stage 1 practitioners and teachers will therefore want to plan learning environments that are rich in mathematical possibilities (see Chapter 2: Enabling environments, page 6). Rich and complex play experiences, supported by sensitive and knowledgeable adults, will enable children to build on their understanding, making and communicating their meanings in a variety of ways.

To promote seamless transitions and continuity for children between Reception and Year 1, headteachers, leaders and managers will need to provide opportunities for Reception and Year 1 teachers and practitioners to learn more about each other's worlds. Identifying opportunities for these teachers and practitioners to spend time in each other's settings will be invaluable.

Continuity into Year 1

The Williams mathematics review acknowledges that transition 'directly affects the young learner in mathematics'. Teachers need to be aware 'of summer-born children who can find the transfer to Year 1 problematic, particularly if the change is abrupt, the environment unhelpful to active children, and the curriculum not flexible enough to take account of a child's stage of development' (DCSF, 2008a). This is also especially important for summer-born children entering Reception classes.

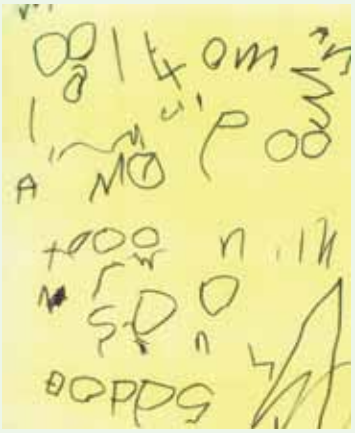
The training resource *Continuing the Learning Journey* (QCA, 2005) focused particularly on the transition from Reception to Year 1. It emphasises the importance of teachers continuing to provide the same sort of experiences and opportunities for play and learning in Year 1 and learning from the assessments from the EYFSP handbook (QCA, 2008). *The Independent Review of the Primary Curriculum* (DCSF, 2009a) also emphasises the importance of play for Year 1, explaining that 'Play is not a trivial pursuit.' Drawing on a robust evidence base, the report highlighted the importance of 'widening the curriculum opportunities for child-initiated and play-based activity' in Year 1 (DCSF, 2009a).

Transition case study – a Year 1 teacher's perspective

Before the children moved into their next class, the Year 1 teacher enjoyed reading the children's 'learning journals' and was impressed by their growing understanding of the many examples of the *children's mathematical graphics* revealed, including the observations of the 'baby clinic'.

'Baby clinic' in Reception

Visiting the local baby clinic, the children watched as health visitors weighed babies and recorded their weights on charts and in books, and listened to discussions about their progress. On their return to school some rich, symbolic play developed spontaneously, supported by a real set of baby-weighing scales their teacher had borrowed.



Over several days, a piece of paper on which one child made marks was added to by others. Their graphics show that:

- they understood that adults use written marks and symbols for specific purposes
- they drew on their knowledge of symbols, including approximations of letters and numerals they had seen

- some children used the initial letter of their name or their age number to stand for what they said.

She decided to develop the play provision for the children who would be coming to her class. To encourage the children to continue to explore their *mathematical graphics*, she also ensured that paper and pens were readily available in every play area and filled a basket with mark-making resources (including large chalks) for their use outside. For the first time she decided to add some mathematical resources to the graphics area and, by moving a cupboard, was able to create a display area at the children's height to encourage them to add their own graphics.

As they started the new school year, the children enjoyed the opportunities that the increased play provision provided and throughout the year frequently chose to explore their *mathematical graphics*, building on their earlier experiences in Reception.

Effective mathematical provision will have similarities in EYFS and Key Stage 1, with a similar range of play provision. The following points will support children to feel confident in their exploration.

Practical pedagogical points supporting smoother transitions

- Provide extended periods of child-initiated play in all settings throughout the EYFS and Key Stage 1.
- Ordering children chronologically in the attendance register will act as a constant reminder of the summer-born.
- Ensure that all settings are rich in resources with potential for mathematics.
- Value children's play, meanings and talk.
- Encourage children to explore their *own* ideas about PSRN.
- Value children's own *mathematical graphics*.
- Prioritise time for transition meetings to provide opportunities for practitioners to share the learning that has taken place to support continuity and seamless transitions for every child.

Transitions in mathematics: recommendations from the Williams mathematics review

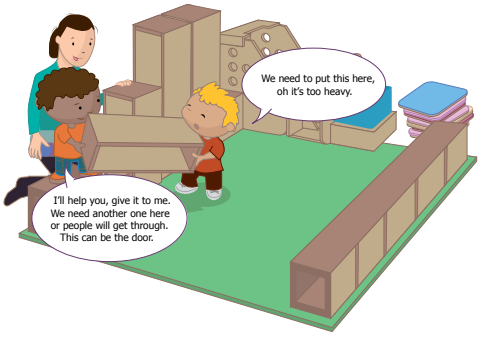
- Successful transition depends on the setting ensuring it is ready to provide appropriately for each child. This requires full account to be taken of the child's accomplishments, and needs to reflect the perspectives of a range of contributors, especially parents.
- Practitioners and teachers must be ready to provide for the individual development and learning needs of each child.
- Familiar approaches to children's mathematical education should be maintained in Year 1.
- Year 1 teachers should be encouraged to increase opportunities for active, independent learning and learning through play, as in the EYFS, to ensure a continuation of positive attitudes to mathematics.
- Mathematical experiences should be threaded across the different areas of learning, in role-play, construction, and in indoor and outdoor learning.
- Children's understanding should be developed using practical resources and should make links with other learning so that mathematics is meaningful and relevant.

(DCSF, 2008a)

8. Examples of problem solving in action from PSRN Essential Knowledge, the e-learning module

The following are extracts from the e-learning course *Problem solving, reasoning and numeracy – Essential knowledge*. They show children engaged with PSRN and in particular problem solving as part of their play. The e-learning course can be found at www.standards.dcsf.gov.uk/nationalstrategies.

Problem solving scenario



Building blocks

The boys are highly engaged in their play. They are collaborating well as they build their structure. They are solving problems as they work out which blocks fit where, how to balance them and how to shift them into place.

Building blocks

The boys were so motivated by their building. Michael was able to calculate that he needed two more mats to fill the interior space left by looking at the area the other mats covered. Through their play the boys are learning new skills and acquiring knowledge.

Maths and stories

The practitioner is stimulating PSRN play through a story and props. She suggests to the children that they could tip out the bag of small dinosaurs and see if they all fit into the blue bucket.

Maths and stories

The children find that the dinosaurs will not fit in their bucket, so they have a problem! The children decide they need a bigger container and remember where they can find one. They are able to fill the larger container with the small dinosaurs. Problem solving happens again and again during play; encourage children to find solutions for themselves.

Problem solving video

Guttering and pipes

The children are playing outdoors with water. They have assembled guttering and pipes in their own ways and are pouring water down into a tub. This child attempts to pour water into a pipe that has a turn at it's neck and the water keeps flowing out again. She persists in her attempts to make water flow down the pipe and eventually realises she needs to turn the neck of the pipe upwards for the water to pour down.

Guttering and pipes

A little while later another child also attempts to pour water down the pipe. He can see the water is flowing back out and is not sure how to make the water travel down the pipe. The first child is watching and tells him that he has to twist the pipe up so that the water can flow down. She articulates her learning and is able to pass this on to another child.

Play-dough cakes

The children are making play-dough cakes. The practitioner models real-world cooking. One child fills his baking tray and tries to put it in the toy oven. He attempts to put the tray in width ways but isn't able to get the tray through the door. He realises the door needs to be opened more fully and gently pushes a child's legs that are in the way.

Play-dough cakes

He still can not fit his tray in and tries turning it lengthwise. It still will not fit and this time he angles it but he tips it too far and his cakes fall on the floor. The practitioner helps him collect his cakes and fit the tray into the oven. Children are able problem solvers and will persist, particularly when motivated by their own purposes and interests.

Problem solving scenario

Imaginative Play

The children use their developing mathematical ideas to solve practical problems. They are sorting and categorising the clothes to suit the weather. Problems arise as they need to match pairs of socks and shoes or find the correct sizes to fit their teddies. What questions or comments could the practitioner use to prompt the children's problem solving, reasoning and numeracy?

Imaginative Play

Perhaps:

'I wonder if I am going to fit all of teddy's clothes into this tiny bag?'

'How many shirts do you think teddy will need?'

'Can you help me find socks? Teddy will need three pairs.'

What other questions or comments can you think of?

Numbers outdoors

The children are on a walk of their local environment. They are taking photographs of things that capture their interests to add to their outdoor area.

Numbers outdoors

The practitioner laminates the photographs the children took and hangs them in sequence along a fence. As children recall their experiences, they can be encouraged to play problem solving games, such as muddling up the sequence of the photos and rearranging in order.

Problem solving video

Pirates' treasure hunt

The children are playing a game of pirates. They hunt for pirates 'treasure'. They make tally marks to find out how many they have gathered altogether. They find they have ten pieces of treasure between them. One of the children knows another way to make ten, other than the 5 + 5 that they have found with their tally marks. He knows 6 + 4 make ten.

Pirates' treasure hunt

The practitioner asks the children how they can check if that is correct. Together they decide to check on their fingers and find this harder than it sounds. They have to consider: which number of fingers should they start with, what they need to do next to see if the sum is correct and how many they need to add on. Checking the sum brings about its own challenge in problem solving.

A parent talking

A father talks about maths when he was a child. He recalls that when he was at school problem solving and reasoning were not encouraged; numeracy, yes, but within the realms of arithmetic. He comments on how he sees his children develop through problem solving. He gives the example of his daughter approaching painting. She works out which brush she wants and experiments with what kinds of marks the brushes can make. He has watched this develop into her being able to paint careful lines and curves.

A parent talking

He sent his son to the Centre because of the great outdoor space, expecting him to run around most of the time. Instead he has watched him develop into 'Mr. Construct It' and sees how his son's imagination knows no bounds. Every day he and another dad wait for their sons at the end of the session and laugh about whose son is going to appear with the biggest box with things stuck all over it that day. The box then gets pride of place at home for a few days.

9. References

Carruthers, E. and Worthington, M. (2003) 'Research uncovers children's creative mathematical thinking', *Primary Mathematics*, Volume 7, Issue 3 (Autumn), pp. 21–25.

Carruthers, E. and Worthington, M. (2003) *Children's Mathematics: Making Marks, Making Meanings*. London: Paul Chapman Publishing.

Carruthers, E. and Worthington, M. (2006) *Children's Mathematics: Making Marks, Making Meanings* (2nd edition) London: Sage Publications.

Carruthers E. and Worthington, M. 'Children's mathematical graphics: young children calculating for meaning' in Thompson, I. (Ed.) (2008) *Teaching and Learning Early Number* (2nd edition) Maidenhead: Open University Press, pp. 127–148.

Carruthers, E. and Worthington, M. (2009) **'**Children's mathematical graphics: understanding the key concept', *Primary Mathematics*, Volume 13, Issue 3 (Autumn).

DfEE (2000) *Mathematical Vocabulary*, London: DfEE.

DfES (2003) *Foundation Stage Profile,* London: DfES. **www.standards.dcsf.gov.uk/nationalstrategies**

DfES (2006) *Guidance Paper: Using and Applying Mathematics,* London: DfES. **www.standards.dcsf.gov.uk/nationalstrategies**

DfES (2007a) *Practice Guidance for the Early Years Foundation Stage*, London: DfES.

DfES (2007b) EYFS card: '*Learning and Development 4.1,* Play and exploration', London: DfES. **www.standards.dcsf.gov.uk/nationalstrategies**

DCSF (2008) Every Child a Talker: Guidance for Consultants

DCSF (2008) Early Years Quality Improvement Support Program

DCSF (2008a) *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools*, Final Report – Sir Peter Williams, London: DCSF. **www.publications.teachernet.gov.uk**

DCSF (2008b) *Mark Making Matters*, London: DCSF. **www.standards.dcsf.gov.uk/nationalstrategies**

DCSF (2009a) *Independent Review of the Primary Curriculum: Final Report,* London: DCSF. **www.publications.teachernet.gov.uk**

DCSF (2009b) *Progress Matters*: *Reviewing and enhancing young children's development,* London: DCSF. **www.publications.teachernet.gov.uk**

Dowling, M. (2006) Supporting young children's sustained shared thinking: An exploration. London: Early Education.

Gifford, S. (2005) *Teaching Mathematics 3–5*, Maidenhead: Open University Press/McGraw Hill.

Gura, P. (Ed.) (1992) *Exploring Learning: Young children and block play,* London: Paul Chapman Publishing.

Pound, L. (2006) *Supporting Mathematical Development in the Early Years* (2nd edition) Maidenhead: Open University Press.

QCA (2005) *Continuing the Learning Journey Inset package,* London: QCA. **www.standards.dcsf.gov.uk/nationalstrategies**

QCA (2008) EYFSP Profile handbook, London: QCA.

Thompson, I. 'What do young children's mathematical graphics tell us about the teaching of calculation?' in Thompson, I. (Ed.) (2008) *Teaching and Learning Early Number* (2nd edition) Maidenhead: Open University Press.

Further reading

Brooker, L. (2008) *Supporting Transitions in the Early Years,* Maidenhead: Open University Press/McGraw-Hill Education.

Carr, M. (2001) *Assessment in Early Childhood Settings: Learning Stories,* London: Paul Chapman Publishing.

Carruthers, E. (1997) 'A numberline in the nursery – a vehicle for understanding children's number development' *Early Years* – (TACTYC), Vol. 18. No.1. Autumn 1997, pp. 9–14.

Carruthers, E. and Worthington, M. (2009) *Children's Mathematical Graphics: Beginnings in Play*, Maidenhead: Open University Press/McGraw-Hill Education.

Carruthers, E. and Worthington, M. (2009) *Understanding and Developing Children's Mathematical Graphics,* Maidenhead: Open University Press/McGraw-Hill Education.

Carruthers, E. and Worthington, M: additional publications:
www.childrens-mathematics.net/references_worthcarruth.htm

Children's Mathematics Network: **www.childrens-mathematics.net**

DCSF (2007) *Creating the Picture*: **www.standards.dcsf.gov.uk/nationalstrategies**

Kress, G. (2007) *Before Writing: Re-thinking the Paths to Literacy,* London: Routledge.

Matthews, J. (2003) *Drawing and Painting: Children and Visual Representation* (2nd edition) London: Paul Chapman Publishing.

Moyles, J. (Ed.) (2007) *Early Years Foundations: Meeting the Challenge*, Maidenhead: Open University Press/ McGraw-Hill Education.

Nutbrown, C. (2006) *Threads of Thinking: Young Children Learning and the Role of Early Education* (2nd edition), London: Sage Publications. (*See chapter 3 on children's developing ideas of mathematical understanding*.)

Pahl, K. (1999) *Transformations: Meaning Making in Nursery Education*, Stoke on Trent: Trentham Books Limited.

Redcliffe Children, (2009) *Food-based Creative Thinking at Redcliffe Children's Centre*, Bristol: Redcliffe Children's Centre.

Vygotsky, L. S. (1978) *Mind in Society*, Cambridge, Mass.: Harvard University Press.

Wood, E. and Attfield, J. (2005) *Play, Learning and the Early Childhood Curriculum* (2nd edition) London: Paul Chapman Publishing.

Worthington, M. (2009) 'Play is a complex landscape: imagination and symbolic meanings' in Broadhead, P. , Wood, L. and Howard, J. (Eds) *Play and Learning in Educational Settings*, London: Sage Publications.

Acknowledgements

The DCSF would like particularly to acknowledge the contribution of Elizabeth Carruthers and Maulfry Worthington.

Special thanks to all the professionals and children involved in the research and consultation for this booklet.

Figures depicting children's work and play taken from Carruthers, E. and Worthington, M. (2006) Children's Mathematics: Making Marks, Making Meaning, (2nd edition). SAGE Publications, London, Los Angeles, New Delhi and Singapore. © E. Carruthers & M. Worthington. Used with kind permission.

Taxonomy (a & b) © Elizabeth Carruthers and Maulfry Worthington, 2006. Used with kind permission.

Photographs and children's work taken from **www.childrens-mathematics.net** © E. Carruthers & M. Worthington. Used with kind permission.

Extract from Carruthers, E. and Worthington, M. (2003) 'Research uncovers children's creative mathematical thinking', *Primary Mathematics*, Volume 7, Issue 3 (Autumn).

Extract from Carruthers E. and Worthington, M. 'Children's mathematical graphics: young children calculating for meaning' in Thompson, I. (Ed.) (2008) *Teaching and Learning Early Number* (2nd edition) Maidenhead: Open University Press.

Extract from Gifford, S. (2005) *Teaching Mathematics 3–5*, Maidenhead: Open University Press/McGraw Hill.

Extract Pound, L. (2006) *Supporting Mathematical Development in the Early Years*. p. 98. © Open University Press. Used with kind permission.

Extract from Thompson, I. (Ed.) (2008) What do young children's mathematical graphics tell us about the teaching of calculation? In *Teaching and Learning Early Number,* Maidenhead: Open University Press, p. 158, (2nd Ed.). Used with kind permission.

Figure depicting child's work (p. 16). Taken from Worthington, M. (2009) 'Fish in the water of culture: signs and symbols in young children's drawing', Psychology of Education Review, Vol. 33, No. 1, March 2009. Reproduced with permission from the Psychology of Education Review. ©E. Curruthers & M. Worthington/The British Psychological Society.

 00861-2009BKT-EN

Audience: Early Years practitioners
Date of issue: 12-2009
Ref: **00861-2009BKT-EN**

Copies of this publication may be available from:
www.teachernet.gov.uk/publications

You can download this publication and obtain further information at: **www.standards.dcsf.gov.uk**

Copies of this publication may be available from:
DCSF Publications
PO Box 5050
Sherwood Park
Annesley
Nottingham NG15 ODJ
Tel 0845 60 222 60
Fax 0845 60 333 60
Textphone 0845 60 555 60
email: dcsf@prolog.uk.com

Published by the Department for
Children, Schools and Families

department for
children, schools and families